JHU-TIPAC 95026

A scale-dependent cosmology for the inhomogeneous Universe

C. W. Kim*, T.H. Lee† and J. Song‡

* †‡*Department of Physics and Astronomy*

The Johns Hopkins University

Baltimore, MD 21218, U.S.A.

and

* †*Department of Physcis, Soong Sil University*

Seoul 156-743, Korea

Abstract

A scale-dependent cosmology is proposed in which the Robertson-Walker metric and the Einstein equation are modified in such a way that Ω_0, H_0 and the age of the Universe all become scale-dependent. Its implications on the observational cosmology are discussed.

*E-mail cwkim@jhuvms.hcf.jhu.edu

†E-mail thlee@jhup.pha.jhu.edu

‡E-mail jhsong@rowland.pha.jhu.edu

The standard Friedmann cosmology is based on the Cosmological Principle (CP) which states that the Universe is homogeneous and isotropic at any given time. While isotropy has been well–established, for example, from the observation of the Cosmic Microwave Background Radiation (CMBR) by COBE [1], homogeneity has been challenged by various observations of large scale structures such as filaments, sheets, superclusters, voids and so on. Moreover, one of the most remarkable consequences of recent galaxy surveys [2] [3] is that the scale of the largest structures in each survey is comparable with the extent of the survey itself, implying the absence of any tendency toward homogeneity up to the present observational limit. Instead, the observed matter density seems to be an increasing function of scale from our underdense neighborhood [4]. Another subject of heated controversy is about the recent measurements of high values of H_0 [5] [6] and their implied age of the Universe which becomes only half the measured ages of $14 \sim 18$ Gyr for the oldest stars and globular clusters.

The above situation has motivated us to propose a cosmological model in which Ω_0, H_0 and the age of the Universe all become scale–dependent, violating the homogeneity in CP. Our model is based on the following two ansatzes. First, we propose that the Universe is described by the metric

$$d\tau^2 = dt^2 - R^2(t,r)(dr^2 + r^2 d\Omega^2), \tag{1}$$

where $R(t,r)$ is the generalized scale factor which is a non–separable function of r as well as t. (If $R(t,r)$ is separable in the form of $a(t)f(r)$, the Robertson–Walker metric, i.e., homogeneity is recovered.) Since homogeneity in CP is violated in this metric, only an observer at the center sees an isotropic Universe. Based on the observations of isotropic CMBR and $\Omega(r)$, we set our position at the center of the metric for mathematical simplicity. The next ansatz is to generalize the Einstein equation in two ways. First, in order to accommodate the r–dependence of the Ricci tensors calculated from the inhomogeneous metric (1), the Einstein equation is generalized to

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -8\pi[GT^{\mu\nu}](t,r)\ . \tag{2}$$

Note that the energy–momentum conservation, $[GT^{\mu\nu}]_{;\nu} = 0$, is naturally obtained in view of the Bianchi identities. Secondly, for an *inhomogeneous* Universe, a generalized stress–energy tensor with the contribution of viscous fluid with heat conduct is introduced as

$$T^{\mu\nu} = \rho u^\mu u^\nu + (p - \zeta\Theta)P^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu \ , \tag{3}$$

where $\zeta \geq 0$ is the coefficient of *bulk* viscosity, $\Theta = 3\dot{R}/R$ and $P^{\mu\nu} \equiv (u^\mu u^\nu - g^{\mu\nu})$ represent the expansion and projection tensor of the fluid, respectively, and q^μ is the heat–flux 4–vector with components $q^\mu = (0, q^{(r)}, 0, 0)$ in the local rest–frame of the *isotropic* fluid.

Even though p_r and $p_\theta (= p_\phi)$ in Eqs.(1) and (2) appear to be different, we assume $p_r = p_\theta$ to avoid shear forces, yielding another constraint on R as

$$\frac{R''(t,r)}{R^3(r,t)} - 2\frac{R'(t,r)^2}{R^4(t,r)} - \frac{R'(t,r)}{rR^3(r,t)} = 0 \ \ , \tag{4}$$

which can be integrated, yielding

$$R(t,r) = \frac{a(t)}{1 - \left[\frac{1+b(t)}{4}\right] r^2} \ \ , \tag{5}$$

where $a(t)$ and $b(t)$ are arbitrary functions of t alone. In [8], it is shown that setting $T^i_i = -(p - \zeta\Theta)$ and T^{01} to be zero with Eq.(5), factorizes $R(t,r)$ as $a(t)f(r)$, which is of no interest to us. Physically, this non–zero T^{01} has been responsible for allowing matter flow from its homogeneous distribution to the present inhomogeneous one. Now, we have two Einstein Field Equations (EFE)

$$\left[\frac{\dot{R}(t,r)}{R(t,r)}\right]^2 = \frac{8\pi}{3}[G\rho](t,r) + \frac{1}{a^2(t)} + \frac{b(t)}{a^2(t)} \ \ , \tag{6}$$

$$\frac{\ddot{R}(t,r)}{R(t,r)} = -\frac{4\pi}{3}[G\rho + 3G(p - \zeta\Theta)] \ \ . \tag{7}$$

In the local Universe$(r \simeq 0)$, $(\dot{R}/R)$ and $(\ddot{R}/R)$ reduce, respectively, to $\dot{a}/a$ and $\ddot{a}/a$, motivating us to interpret $a(t)$ as the scale–factor of the local Universe with the modification $b(t)$. Assuming the following form of the $\zeta\Theta$ term

$$8\pi G\zeta\Theta = \left[\chi^{(0)}(t)\frac{\dot{a}_0}{a_0} + \chi^{(1)}(t)\frac{\dot{R}}{R}\right]\frac{\dot{R}}{R} \ , \tag{8}$$

the behaviors of ρ, p and T^{01} can be determined by Eqs.(2) and (5) as

$$\frac{8\pi}{3}G\rho = \left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2} - \frac{b}{a^2} + 2\left(\frac{\dot{a}}{a}\right)D(t,r) + D^2(t,r) \tag{9}$$

$$8\pi Gp = \frac{1}{a^2} - 2\frac{\ddot{a}}{a} + \chi^{(0)}\frac{\dot{a}_0}{a_0}\frac{\dot{a}}{a} + [\chi^{(1)} - 1]\left(\frac{\dot{a}}{a}\right)^2 + \frac{b}{a^2} + \left(\chi^{(0)}\frac{\dot{a}_0}{a_0} + (2\chi^{(1)} - 6)\frac{\dot{a}}{a} - 2\frac{\ddot{b}}{\dot{b}}\right)D(t,r) + [\chi^{(1)} - 5]D^2(t,r) \tag{10}$$

$$8\pi GT^{01} = \frac{\dot{b}r}{a^2} \ \ , \tag{11}$$

where $D(t,r)$ is given by

$$D(t,r) \equiv \left(\frac{\dot{b}r^2/4}{1 - \left[\frac{1+b}{4}\right]r^2}\right) \ . \tag{12}$$

From the observed *increase* of the present mass density, we assume $\dot{b} > 0$ in the matter–dominated era. The homogeneous Universe in the early radiation–dominated era can be obtained in this model by requiring $\dot{b} = 0$. In the following, we restrict ourselves to the matter-dominated era. Now, the sign of $[1 + b(t)]$ term becomes crucial because of $D(t,r)$ terms in ρ and p. The spatial curvature, $R^{(3)} = -[1 + b(t)]/a^2(t)$, suggests that $[1 + b]$ be positive in order to explain the locally *open* Universe, implying an apparent singularity at $r = 2/\sqrt{1+b} \equiv r_H$. This singularity is, however, spurious one as $r = 2GM$ singularity in the Schwartzschild metric, because $R^{(3)}$ is finite at $r = r_H$ as at $r = 2GM$. Therefore, the point $r = r_H$ is interpreted as an event horizon.

Let us discuss physics at this event horizon. First, we have a specific relation between p and ρ at $r = r_H$. We have, from Eqs.(9) and (10),

$$\lim_{r\to r_H}\frac{Gp}{G\rho} = \frac{\chi^{(1)}(t) - 5}{3} \equiv \gamma - 1 \ . \tag{13}$$

The so–called gamma–law equation of state ($0 \leq \gamma \leq 2$) can be satisfied by restricting the value of $\chi^{(1)}$ in the range [2,8]. Since $(p - \zeta\Theta)$, not p, is involved in Eq.(7), dynamics at $r = r_H$ is described by $[(p-\zeta\Theta)/\rho]_{r=r_H} \to -5/3$, implying $\ddot{R} > 0$ (see Eq.(7)). This ($\ddot{R} > 0$)

is the very condition of the generalized inflation [7], suggesting a picture of the Universe which is inside an *expanding* shell with infinite mass density.

In this model, cosmological quantities such as the expansion rate, Ω and the age of the Universe are all functions of t and r. The expansion rate and Ω, in this model, are

$$H(t,r) \equiv \frac{\dot{R}(t,r)}{R(t,r)}, \tag{14}$$

$$\Omega(t,r) \equiv \frac{\rho(t,r)}{\rho_c(t,r)} = 1 - \frac{1+b(t)}{a^2(t)} \frac{1}{H^2(t,r)} \ , \tag{15}$$

where ρ_c is defined by $H^2 \equiv 8\pi G\rho_c/3$. We must caution the reader that Eqs.(14) and (15) cannot be observed at (t,r), for every observation is based on the light propagation given by

$$dr = -\frac{dt}{R(t,r)} \ , \tag{16}$$

whose solution is denoted by $r(t, t_{received})$, explicitly showing the boundary condition $r(t = t_{recieved}) = 0$. In [9], Eq.(16) is numerically solved with the boundary condition $r(t_0,t_0) = 0$ by specifying the functional forms of $a(t)$ and $b(t)$ under the assumption that $a(t)$ resembles the scale factor of the Friedmann cosmology with $k = -1$ and $b(t)$ is a small modification. Fitting them in the form of $r(t,t_0) = \delta[t_0^\kappa - t^\kappa]$ gives an expression for the redshift z as $1 + z = (t/t_0)^{\kappa-1}$. With the definition of the luminosity distance d_L as $d_L \equiv rR(t,r)(1+z)$, the redshift–luminosity distance relation is presented in [9]. The numerical calculations show that the linear relationship between z and d_L for nearby objects ($z \ll 1$) remains intact in this model also, and the overall Hubble diagram for a locally *open* Universe with $\Omega(t_0, r \simeq 0) = 0.1$ is similar to that of the *flat* Friedmann cosmology. What, then, is the physical meaning of the observed increase of Ω in this model? Every direct information obtained by light signal reaching us right now is about the past, including $\rho(t)$, where $t < t_0$. We deduce $\rho(t_0)$ from the observed $\rho(t)$ in the framework of the Friedmann cosmology, that is, using $\rho(t)/\rho(t_0) = S^3(t_0)/S^3(t) = (1+z)^3$. What is considered to be Ω_0, therefore, is in this model,

$$\Omega^{obs} = \frac{\rho^{deduced}(t_0)}{\rho_{c,s}} = \frac{\rho(t, r(t))}{\rho_{c,s}(1+z)^3} \ . \tag{17}$$

With the numerical solution described below Eq.(16), $\Omega^{obs}(z)$ is plotted in [9], showing that $\Omega^{obs}(z)$ is indeed an increasing function of z.

Finally, we discuss the age of the Universe, which is in crisis in the framework of the Friedmann cosmology due to the measurements of high values of H_0. The age of the Universe in this model is also r–dependent as $H(t,r)$ and $\Omega(t,r)$. Since the information of $t_0(r)$ comes through the light propagation with finite speed, however, the measurement of the *present* age of the Universe at r can only be accomplished in the future. That is, only the *local* age of the Universe is the *true* age of the Universe, appropriate for comparison with the age of the stars in our galaxy. Defining x as $a \equiv a(t)/a(t_0)$, Eq.(6) at $r \simeq 0$ yields

$$\dot{x} = \sqrt{\frac{8\pi G \rho(t, r\simeq 0)}{3}\frac{a^2(t)}{a^2(t_0)} + \frac{1+b(t)}{a^2(t_0)}} \ . \tag{18}$$

Assuming that the whole history of the Universe is dominated by the matter–dominated era, when T^i_i and T^{01} are relatively small compared with ρ, we have $\overline{\rho}(t) \sim 1/a^3(t)$ from the energy–momentum conservation. Then, Eq.(18) yields

$$\overline{t}_0 \simeq \frac{1}{\overline{H}_0}\int_0^1 \frac{\sqrt{x}}{\sqrt{\overline{\Omega}_0 + (1-\overline{\Omega}_0)}} dx \simeq \frac{0.9}{\overline{H}_0} \ , \tag{19}$$

where we have used Eq.(14) at $r \simeq 0$. Note that the true age of the Universe is determined by the *local* values of Ω_0 and H_0. For example, with $\overline{\Omega}_0 = 0.1$ and $\overline{H}_0 = 50$ Km/secMpc, the age of the Universe in this model is 18 Gyr, which can easily accommodate the observed age $14 \sim 18$ Gyr of the oldest stars and globular clusters in our galaxy.

In conclusion, the scale–dependent cosmological model proposed here can explain, at least qualitatively, the observed increase of the mass density, the age problem and a conflict among current measurements of the Hubble parameters.

REFERENCES

[1] J.C. Mather *et al.* 1990 AP.J. **326** L37; G.F. Smoot, 1994 AP.J. **396** L1 ; E.L. Wright *et al.*, 1993 AP.J. **396** L13.

[2] T.J. Broadhurst, R.S. Ellis, D.C. Koo and A.S. Szalay, 1990 Nature **343** 726.

[3] G. Vettolani, *et. al.* 1993, in the Proceedings of Schloss Rindberg Workshop.

[4] D. Schramm, Phy. Rep. **227** (1993) 13.

[5] M.J. Pierce, *et. al.* Nature **371** (1994) 385.

[6] W.L. Freedman, *et. al.* Nature **371** (1994) 757.

[7] F. Lucchin and S. Matarrese, Phys.Lett. **B164**(1985)282.

[8] C.W. Kim and J. Song, JHU-TIPAC 95016.

[9] C.W. Kim, T.H. Lee and J. Song, JHU-TIPAC 95023.